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Nonlinear finite element bending analysis of Cold-Formed Steel of 'Z' section beams

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Abstract

Cold-formed steel Z section beams are widely used as the secondary structural members in buildings to support roof and sheeting or side cladding. In general, cold-formed steel sections are slender elements and have open and/or asymmetric cross-sections, where centroid and shear centre do not coincide. Therefore, during bending the section twists and deflects in both lateral and transverse direction resulting in the reduction of stiffness. Currently, design codes for this section use classical bending theory, which assumes load-deflection relationship to be linear and does not consider the effect of bending-torsion in the calculation of deflection. In CFS members for serviceability and deformation consideration, it is essential to consider the loss of stiffness due to the use of thin elements. Hence, this paper presents numerical investigations using the finite element method to check the accuracy of the linear analysis. A geometrical nonlinear analysis using ANSYS is performed on Z-beams with different section size and thickness to study the sectional deformation on its performance under bending. The obtained deformation verified the effect of bending-torsion in CFS cross-section. The predicted deformation characteristic curves at mid-span are examined and compared with linear solutions and maximum deflection predicted by serviceability limit states. The comparison showed that there is a significant difference in displacement between FEA nonlinear solutions and linear solutions, as well as serviceability deflection.

Introduction

In steel construction, there are primarily two categories of steel used, namely, the hot-rolled and cold formed sections. Hot-rolled steel sections are formed by rolling under significantly elevated temperatures (i.e. above 900° C), while the other type; Cold-Formed Steel (CFS) are shaped into various structural members at ambient temperature by cold-forming steel sheets or plates employing of roll-forming or press-braking operations. CFS sections have been manufactured and used in construction for more than a century. However, in recent years, higher strength materials and development in structural applications have led to a significant growth of CFS in industrial, residential, agricultural and commercial applications relative to the conventional hot-rolled steel. The concept of CFS elements is to use shape rather than the thickness to support the load. Due to ease of manufacturing at ambient temperature, cold-formed elements are available in a variety of complex structural shapes to fit the demands of optimised design. Cold-forming operations increase the yield point and tensile strength by about 20% to 40%, depending on the type of manufacturing. Furthermore, CFS members have the following advantages:

- High strength to weight ratio: CFS sections are generally lighter and stronger when compared to hot-rolled counterparts.
- Ease of prefabrication and mass production: Cold-forming makes manufacturing of complex configuration economic and flexible in quantities.
- Fast and easy installation and erection making them a cost effective material
- Ease of handling and transportation.

Due to these advantages, the use of CFS members has increased recently in the building industry relative to traditional hot-rolled steel counterparts. The use of CFS structural profiles in building construction started around the 1850s in the United Kingdom and the United States. However, they were not widely used in buildings until 1940, due to limited information about their structural behaviour (Yu and LaBoube, 2010). The CFS sections are most commonly used in building applications, as secondary cladding, purlin systems and the intermediate members in a roof system to support the corrugated roof or wall sheeting and transmit the force to the main structural frame. More recently, these are being increasingly used in primary applications, as beams and columns of industrial and house system. Among various types of modern CFS wall and roof system, Z section purlins are one of the most popular due to their high structural efficiency with the simple and effective installation. Hence, in this study emphasis given to Z section to understand their structural behaviour for efficient and engineered design and construction of modern roof and wall system. The most commonly utilized shapes are C and Z-sections, which may be plain or have stiffened lips and section thickness typically ranging from 1 mm to 3 mm, while the section depth range from 100 to 350 mm. Common yield strengths are 280 and 350 N/mm^2 but recently, section with yield strength up to 450 N/mm^2 can be found for improved load carrying capacities.

With the rapidly rising usage of CFS members, studies on the understanding of their structural behaviour have increased significantly since the first specification of CFS design was established in 1946. The central focus of these research efforts was mainly on the understanding and dealing of instability phenomena such as local, distortional, lateral-torsional buckling and their interactions. These buckling modes may occur even before the cross-section yields resulting in the ultimate strength of the compression member. The characteristics of CFS members such as the low torsional stiffness, the high slenderness and the geometric imperfections are some of the main causes for their high susceptibility to buckling phenomena (Yu and LaBoube, 2010). It should be further noticed that the CFS cross-section walls are generally slender i.e. thickness is usually less than 3 mm and that the most cross-sections are open and/or asymmetric which is in the case of Z section; in other words, the shear centre and the centroid of the section does not coincide. Therefore, when the section is bent about its major axis, it develops a tendency to twist and/or deflect in both the lateral and transverse directions and this effect of bending-torsion results in the reduction of section stiffness. When members are optimized for strength, the stiffness of section becomes an important consideration, as the resulting stiffness often controls the design, particularly in the case of beam applications. A review of various modern codes of practice for design of CFS structures reveals that the Serviceability Limit State (refer to conditions of the structure in use, including deformation, cracking and vibration) is generally governed by the stiffness of the structure calculates the deflection of structure based on classical bending theory. This theory assumes that the relationship between load and deflection is linear; hence, the stiffness of the section remains constant during bending. Therefore, in order to take in the account of bending-torsion of the section, there is a need to consider nonlinear analysis in the design of Z section beams to ensure safe and better performance in applied structural applications.

The majority of studies in the literature emphasise further the structural behaviour of CFS member by the means of numerical methods accounting with just one profile. The present paper formulates nonlinear FEA model to study the effect of torsion on the deflection of Z section profiles. To understand the characteristics, design methods and numerical methods of CFS structures for accurate modelling and analysing their flexural behaviour a thorough literature review was undertaken. Followed by literature study, finite element model was established for half span using ANSYS program. The model utilizes a 4-node isotropic shell element and considers the geometric nonlinearity. After applying the displacement boundary and load condition, a large deformation solution type was selected to achieve nonlinear solution. The results were compared and discussed with the conventional linear solution to check their accuracy and lastly, conclusions were drawn for CFS-Z section beams for efficient and engineered design and construction of modern roof and wall system, as well as other structural applications CFS-Z section beams.

Aim and objectives

The fundamental aim of this report is to study the effects of sectional deformation on the performance of Z section profile of different dimensions and thickness under bending in order to check the accuracy of the linear analysis. This aim was achieved by following and achieving the following objectives:

- By developing a numerical model using finite element method for nonlinear analysis of Z section beams.
- By the evaluating the load versus deformation characteristics of different profiles under flexure loading.
- By comparing linear and nonlinear solution obtained from analysis.
- To examine the accuracy of the linear solution currently used in design manuals.

Literature review

In order to understand flexural behaviour of CFS members and why there is need of this study, a thorough literature review was undertaken. This literature review includes review of the characteristics, design methods, experimental methods and numerical methods to analyse and accurate modelling of CFS sections followed by a summary which presents main findings and gaps in the literature.

Main characteristics of Cold-Formed Steel members

CFS members have various advantages in the structural application in contrast with traditional hot-rolled steel. However, they have a variety of instability problems, as they are usually thinner than hot-rolled sections. Moreover, the cold-forming process often creates structural imperfection and residual stresses different from those of hot-rolled members. Therefore, design specifications are required especially for cold-formed steel member.

Cold-forming process

Generally, CFS members are manufactured using two different methods: roll-forming or brake pressing. Roll forming is a continuous process in which pairs of rolls shapes flat sheet or coiled into profiles or sections. It is often used in the production of large quantities of required shape. Press braking is used to manufacture the different variety of sections in small production runs. It is much slower process than roll forming in which sections are produced by pressing sheets, forming one or two bends at a time. Steel strip, which forms the basis of CFS, is available in various grades in flat sheet or coil which represents different structural and forming qualities. The quality and specification of these sheets are given in various standards. Both of these manufacturing methods will cause different geometric imperfections and residual stress distribution in the steel members (Schafer and Pekoz, 1998b).

Material properties

Material properties play a crucial role in the performance of structural steel members. Hence, it is important to find out the mechanical properties of steel structural member for the purpose of design. Besides, mechanical properties are greatly affected by temperature, the designer must give special attention to extreme conditions below -30°F (-34°C) and above 200°F (93°C) (Yu and LaBoube, 2010).

From a structural perspective, primary mechanical property of steel which is used to describe its behaviour is the stress-strain graph. Generally, there are two types of stress-strain curves for steel**.** They are the sharp-yielding and gradual-yielding type. The behaviour of hot-rolled steel is usually illustrated by the sharp-yielding curve, which is depicted by an initial linear elastic region and a rapid transition to a plastic or inelastic phase defined by horizontal curve followed by strain hardening at high strains. Cold-formed steel shows gradual yielding; rounded knee is formed after the elastic region, and as there is no well-defined yield stress, the 0.2% proof stress is generally used to define a value of its yield stress. Generally cold-formed steel doesn’t show any plastic region, and strain hardening occurs immediately after yielding. The yield stress of cold-formed steel increases with the amount of cold work executed to produce the section shape. The mechanical properties of the steel sections are mostly affected by the cold forming work, particular in the regions of bends. In these regions, the material ultimate tensile strength and yield strength are enhanced. The ultimate tensile strength of steel sheets in the cold-formed section has a little straight relationship to the design of such members except bolted and welded connections. The load-carrying capacities of cold-formed steel flexural and compression members are restricted by yield point or buckling stresses that are less than the yield point of steel. Studies indicate that the effects of cold work on formed steel members depend largely upon the spread between the tensile and the yield strength of the virgin material (Yu and LaBoube, 2010).

Geometric imperfection and residual stresses

Geometric imperfections are expressed as the deviation of a member from its perfect geometry. These include bowing, warping and twisting, as well as local deviations. Global and local distortion can occur due to the manufacturing process of the plates and as results of accidental impacts during the transportation. The ultimate strength of cold-formed steel section is influenced considerably by these initial geometric imperfections. Chou et al. (Chou et al., 2000) performed a buckling analysis using the finite element method and mentioned that the initial geometric imperfections must be considered to achieve the ultimate load capacity of structure accurately. Schafer and Pekoz (1998b) presented a series of experiments on the channel sections to measure the geometric imperfections. Maximum magnitudes of the local geometric imperfections were recommended in Schafer and Pekoz (1998b). Since the initial imperfections influence the ultimate load capacity of CFS sections it is important to consider it in finite element model and analysis. Several computational modelling methods have been suggested by Schafer and Pekoz (Schafer and Pekoz, 1998).

Residual stresses are locked in stresses and strains in structural steel in their unload state. Residual stresses in hot rolled section are generally due to the result of uneven cooling, while in CFS member are induced mainly through the cold forming processes such as coiling, uncoiling, cold bending to shape, and straightening of the formed member. The cold forming process at corner regions and intermediate stiffeners cause plastic deformation which is unable to recover due to the new permanent shape, which in result leads to locked longitudinal and transverse residual stresses and strains in the section which can cause premature yielding that can further influence the ultimate load capacity of the elements. Residual stresses in CFS members are considered as the summation of flexural and membrane components. Schafer and Peköz (1998b) concluded from their experimental results that average membrane residual stresses are reasonably small for rolled-formed and pressed braked sections when compared with the flexural residual stresses and can be ignored. Moen et al. (2008) through analytical models showed that the variation of residual stress through the thickness is rather complex and nonlinear similar finding was made by Shafer and Pekoz earlier in experimental analysis.

Structural behaviour of CFS beams

There have been numerous investigations on the structural behaviour of CFS beam over the last decades. These studies were largely focused on the instability phenomena such as local, distortional and lateral-torsional buckling of CFS beams of individual sections. Due to their nature, the cross-section of thin walled CFS members is commonly open, and they have a relatively low flexural rigidity compared to heavy steel sections. In conjunction with subsequent advantages from the cold-forming process like high strength to weight ratio, flexibility and profile precision, CFS member have a different type of deformation under transverse loading, particularly when inadequate restraint is applied to the flexural member. Other characteristics of CFS members such as the low torsional stiffness, the high slenderness, and the geometric imperfections are some of the main causes for their high susceptibility to buckling phenomena (Yu and LaBoube, 2010). It is widely known that CFS section beam, when subjected to bending moment may exhibit local, distortional and global buckling.

Local buckling is mainly prevalent and is characterized by the relatively short-wavelength buckling of individual plate elements whereas Distortional buckling involves both translation and rotation at the compression flange/lip fold line of the member. Distortional buckling occurs as a result of distortion of a portion of the cross-section and predominately rigid response of the flange/lip. Lateral-torsional buckling or global buckling occurs when the cross-section buckles without distortion (Yu and Schafer, 2002). All of these buckling modes are the most interesting and complex subjects within this research field. Apart from them, interactive buckling modes are the ones most prevalent in the CFS flexural members. These buckling modes are mainly accountable for the ultimate strength of the compression member as they can occur even before yielding of cross-sectional parts.

Design methods or design methods of CFS beams

Currently, the Effective Width Method (EWM) and the Direct Strength Method (DSM) are two major approaches considered in design specifications of CFS members. EWM was initially introduced by Von Karman et al (1932) and was later modified by Winter (1947) for CFS. It has now been well established and adopted in design specifications such as Eurocodes (EN1993-1-3, 2006), American Iron and Steel Institute (AISI, 2007) and Australian/New Zealand Standard (AS/NZS, 2005). DSM a relatively recent concept was developed by Schafer and Peköz (1998a), has been included as an alternative procedure in American Iron and Steel Institute in 2004 (AISI, 2007) and Australian/New Zealand Standard for CFS strength determination.

EWM

The basic idea of the EWM is that local pate buckling leads to reductions in the effectiveness of the plates that comprise a cross-section in isolation; furthermore, it can be better understood as an idealized version of equilibrium in an effective plate under a simplified stress distribution vs. the actual plate with a nonlinear stress distribution due to buckling (Schafer, 2008). For a uniformly compressed rectangular plate the stress distribution is uniform prior to buckling load. As the plate starts to buckle, it deflects laterally and experiences a stress gradient, with the central area unable to support any additional load although portions close to the supported edges continue to carry increasing load. This stress distribution for the post-buckling stage is complicated in nature and difficult to calculate in practice, since the variation for individual members can be significantly different. Instead, this loss in plate effectiveness can be considered as an approximate means to represent the equilibrium of stress applied on the partial width of the plate, particularly as ‘ the effective width’.

In the design codes integrated EWM includes effective width for both web and compression flanges when the section is considered under local buckling. In Eurocode 3, the theory of EWM extended to a ‘reduced thickness’ of edge or intermediate stiffeners for the effect of distortional buckling. However, inter-elements i.e. between the flange and the web, are not taken into account since EWM considers each element separately hence this method is not accurate for complex sections. Moreover, with the development of complex shapes such as additional folds and stiffeners added to the sections, determining the effective width of the section becomes increasingly more complicated.

DSM

In the meantime, an alternative method developed by Schafer and Peköz (1998a) examined the elastic buckling solutions for the whole cross-section rather than the individual elements, and strength curves for the entire member called Direct Strength Method (DSM). DSM assumes that local buckling behaviour can also be predicted by the elastic buckling stress of the entire section with a suitable strength design curve for local instability. In DSM, all of the elastic instabilities for the gross section i.e. local (Mcrl), distortional (Mcrd), and global buckling (Mcre) are determined and also the moment (or load) that causes the section to yield (My) is determined. Therefore, the strength (Mn) can be directly determined by Mn = f (Mcr`, Mcrd, Mcre, My) (Schafer, 2008). Moreover, DSM delivers an equivalent level of accuracy and simple calculation procedure in comparison with EWM for predicting CFS member capacity, particularly for complicated cross-section geometry, since it does not require calculating the stress for each individual element within the member.

Both EWM and DSM provide a simplified solution to an initially complex nonlinear problem with CFS member to allow engineers have a working model to design without the requirement of testing every individual member. However, it is important to recognize that neither EWM nor DSM is totally correct as the formulae stated are based on limited experimental test data. With the substantial development of new cross section and dimensions of CFS members over recent years, it is suggested by Schafer (2008) that a completely nonlinear computational model may be the ideal solution in the long term for structural analysis of CFS members. Therefore, in this research, emphasis will be drawn to the numerical modelling using Finite element method to study the sectional deformation behaviour of CFS Z section members.

Experimental methods

Traditionally, experimental analysis has been more widely used for the design of cold-formed sections than for most other building components. These methods have allowed the researcher to directly investigate the bending behaviour of CFS members. As analytical methods have enhanced, experimental testing is now used less frequently. However, it is still significantly used for cases involving the interaction of CFS components with other materials or structure. Unfortunately, due to a large number of CFS shapes, sections, and material grades, it is not economical and it is also very time consuming to experiment every possible arrangement. Therefore, numerical methods such FEA are becoming widespread for analysing CFS members, which are discussed later section.

Numerical methods

With advancement in computers and software in the past few decades, numerical analysis has emerged relatively recently. The majority of research have shown an obvious advantage that numerical method can offer to the traditional experiments in studying thin-walled steel structures, especially for extensive parametric studies (Yu and Schafer, 2007; Trahair, 2002). The increasing amount of complexity in geometry and diversity of CFS sections makes numerical method an efficient tool for analysing.

Typical numerical methods used in the study of CFS members include Finite Strip Method (FSM), Finite Element Method (FEM) and the relatively new approach of

Generalized Beam Theory (GBT). This dissertation focuses predominantly on research methods using finite element modelling. FSM and GBT have also been shown as accurate methods, which are explained in the following sub-sections:

FSM

The FSM was firstly proposed by Cheung (1976) and was later developed by Lau and Hancock (1986) and Schafer (1997) to analyse the stability of CFS sections. The basic principle is to mesh the section in the cross-sectional direction such that it is divided into a number of strips. FSM assumes that the deformation pattern of a model in one direction can be estimated analytically and model could be discretised into strips to study individual sections under longitudinal stress through a shape function. It has lesser degrees of freedom requiring less computing time and memory compared with the FEM. However, selection of the shape function for longitudinal displacement field is of great significance. Schafer (2006), Hancock and Rhodes (2002) have developed the FSM over the years to cover a range of CFS sections subjected to flexural loading. The nonlinear analysis can be performed using FSM but not efficiently and particularly with the material nonlinearly type of problems (Friedrich, 2000). Limited numbers of computer programs have been developed based on the FSM, for example, Thin-Wall and CUFSM (Schafer, 2006). However, Thin-Wall examines only the simply supported end conditions for columns, while the CUFSM (Schafer, 2006) allow calculating the elastic buckling modes and capacities of thin walled sections with arbitrary boundary conditions.

GBT

The generalised beam theory (GBT), originally developed by Schardt (1994) is generally considered an extension of the traditional prismatic beam theory that accounts for cross-sectional deformation. The method was later extended by Davies et al (1994a, 1994b) to examine the stability behaviour of both linear (first-order) and nonlinear (second-order) thin-walled steel sections. In GBT, the deformed configuration of a member is expressed in terms of pre-established cross-sectional deformation modes with varying amplitudes along the member axis. In this way, GBT is able to analyse the buckling modes into a linear relationship of cross-sectional deformation modes. It is able to take account for the geometry imperfection and nonlinearity. The ability to separate different buckling modes for a range of loading, e.g. bending or axial compression makes the method especially amenable to design methods.

In particular, Camotim et al. (2010) have extensively developed GBT, so that it can be applied to different types of analysis, boundary and loading conditions and materials. With a few exceptions, the material models adopted in all these works were always linear elastic. A physically non-linear GBT formulation was first developed by Gonçalves and Camotim (2011) in the terms of elastic–plastic bifurcation analyses. They also proposed GBT beam finite elements (BFE) based on the J2-flow plasticity theory and aimed at performing member first-order and second-order elastic–plastic analyses. It has been widely shown that GBT forms a powerful, elegant and illustrative tool to solve structural problems involving cold-formed steel sections (Camotim et al, 2006).

FEM

Advances in the field of computer-aided engineering have led the use of finite element analysis (FEA) in thin-walled steel structures increasing rapidly in the last decade due to its higher accuracy for analysing numerically. In FEM, the domain is divided into smaller discrete sections called finite elements, which are interconnected at nodal points resulting in set of simultaneous algebraic equations. FEM then solve these equations using partial differential equations by replacing continuous functions with piecewise approximations defined on polygons (Yu and Schafer, 2005). The distribution of elements within the domain called mesh is geometry dependent as well as on the precision of the solution. Therefore, FEM solves by reduces the problem into a set of linear equation using polynomial approximations.

Displacement based finite element method is usually used in structural analysis, in which a set of functions are established such that they distinctively define the state of the displacement within each element in terms of its nodal values. The number of degrees of freedom is now finite and displacement for each element is approximated using shape function and the nodal values. Hence, displacement at each nodal point is calculated by solving these equilibrium equations (Zhao, 2014).

Generally, FEA is consists of three fundamental steps:

1. Pre-processing - At this stage model is developed and its geometry is meshed to divide into finite elements, and material properties, loads, and boundary conditions are applied.

2. Analysis – The dataset prepared at earlier stage is applied to generate matrix equations for each element, which are then assembled and solved numerically for the type of analysis (e.g. static or dynamic) selected. The form of equations is always;

$$\mathbf{K_{ij}u_j = f_i}$$

   Where, u and f are the displacement and applied load at the nodal points and the formation of the K matrix is dependent on the type of problem being chosen.

3. Post-processing – The obtained results are validated and examined in terms of displacement and stresses at a discrete position within the model using graphical representation.

The finite element based commercial software ANSYS and ABAQUS are commonly used to predict the structural behaviour and ultimate strength of CFS structural members. Schafer and Pekoz (1998b), Sivakumaran and Rahman (1998) and Schafer et al. (2010) summarized the aspects to ensure the accuracy and reliability of the FEA of CFS members, should be noted:

- Proper element mesh
- Non-linear material and geometric properties
- Residual stresses
- Initial geometric imperfections
- Boundary conditions and loading algorithm

<u>Analysis</u>

Following the pre-processing phase, the finite element model can be analysed using a finite element program such as ANSYS. During this process, ANSYS program performs a sequence of statistical tasks using a system of simultaneous equations generated from the model and resultant nodal values are stored. The program offers types of various types of equation solvers subject to the type of analysis. The type of analysis could be linear or non-linear and is specified at the beginning of the solution process. The structural behaviour of a model could be analysed based on its linear or nonlinear response to external forces.

Nonlinear analysis

In order to understand the nonlinear system, it is essential to define linear systems first. A linear system is defined such that input-output the relationship is linear. For instance, in structures, when an applied load is doubled, the displacement will also be doubled. Therefore, the method of superposition can be used to solve the linear system again when applying different magnitude of the load. Furthermore, a structural linear system can be illustrated by the flow of physical quantities within them. When the load is applied to the system, local stresses are produced against the globally applied load. In an elastic system, deformation of shape leads to stresses and consequently strains are developed. Strains are integrated at each point to generate the displacement in the global level. When all relationships between loads, stresses, strains and displacement are linear, then the system is called linear. If any of these physical quantities is not linear, them the structural system becomes nonlinear (Kim, 2015).

Linear analysis delivers a fast solution with less amount of modelling time. However, small displacement and rotations in the linear of the material behaviour will give accurate solutions. In practice, structures show nonlinear behaviour within their actual loading ranges, principally due to changing status including contact, geometric nonlinearities and material nonlinearities (Kulatunga and Macdonald, 2013).

Several common structural features exhibit nonlinear behaviour which is status-dependent. For instance, a tension cable is either slack or taut and roller support is either in contact or not. Therefore, status changes are either related directly to load or are determined by other external cause. When the structure is undergoing large deformation, its changing geometric configuration can lead to nonlinearity called as geometric nonlinearity. It is characterized when deformed and undeformed state of the structure is significantly different; a fishing rod in use is an ideal example. Material nonlinearity occurs due to the steel nonlinear stress-strain relationship. Hyperelastic, elastoplastic, or viscoelastic materials are examples of nonlinear materials.

ANSYS uses the “Newton-Raphson” approach to solve nonlinear problems in which the load is subdivided into a series of load increments applied over several load steps. The Newton-Raphson method evaluates the difference between the restoring forces and loads, then using this evaluation program perform a linear solution and checks for convergence. This iterative procedure continues until the convergence criteria is satisfied (ANSYS, 2010). Figure 1 shows the use of Newton-Raphson equilibrium iterations in the nonlinear analysis.

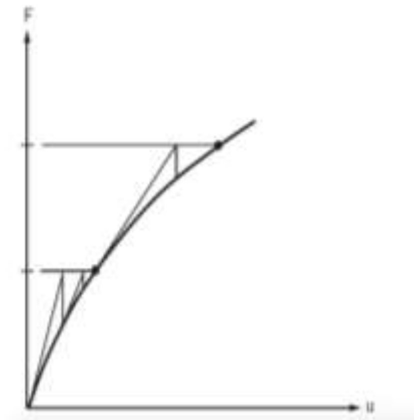

Figure 1: Newton-Raphson Approach

Nonlinear analysis is crucial to the FEA of thin-walled CFS members. Bakker and Pekoz (2003) evaluated the common errors in non-linear analysis and presented methods for overcoming these errors.

Related research and FEA studies

During last decades, numerous investigations have been conducted on the structural behaviour of CFS sections. These studies were mainly focused on the instability of CFS beams of individual section such as zed sections (Chu et al., 2006), sigma sections (Li, 2009), channel sections (Ren et al., 2006), and lipped channel (C) sections (Wang and Zhang, 2009). It is noticed that these cross-sections are opened or asymmetric and may go under large rotation before failure. On the other hand, with the objective of expanding the load capacity of CFS members, many researches have been performed with the issue of the optimum design of CFS beams. For example, Lee et al. (2005) used the micro Genetic Algorithm to calculate the optimization problem of the CFS channel beam section, Karim and Adeli (1999) described the neural network model for the use in global optimum design of CFS omega sections beams, and Tian and Lu (2004) optimized the steel C-channel with or without lipped flanges.

However, at the same time also, the most of the analysis performed on the CFS sections to study sectional performance were by use of FEM. Sivakumaran and Abdel-Rahman (1998) developed a finite element model to investigate the post-local buckling behaviour of cold-formed stub column of channel section. The model predicted the post-local buckling behaviour of non-perforated and perforated CFS members subjected to axial compression.

More recently, Haidarali and Nethercot (2011) developed FE models of CFS beams incorporating geometrical and material nonlinearity to investigate the local and distortional buckling behaviour of Z section with various edge stiffener details. It was found that by varying the lip-to-flange ratio, a transition between local, distortional and local/distortional buckling can be observed. Furthermore, results from FE model were compared with design capacities obtained from Eurocode 3 and design capacities were found to be, on average 10% conservative. A modified buckling factor suggested as an improvement to current Eurocode method. Georgieva et al (2012) used the finite element method to study built-up double-Z member in bending and compression. FEA considering the initial imperfections and stiffness of connections was used to study the various buckling effects that shape the response of slender thin-walled members. The model which was experimentally validated shows that a non-linear FEA can predict the member behaviour in terms of failure mode, ultimate load, local strain, overall stiffness and yield line pattern in the CFS profiles.

Generally, numerical studies on CFS members subjected to compression are more common than the case of bending. Loughlan et al. (2012), Borges Dinis et al. (2007) and Schafer et al. (2010) demonstrated that FE models can account for the buckling behaviour of CFS column sections when subjected to axial compression. Whereas for bending, Yu and Schafer (2003, 2006) and Wang and Zhang (2009) performed the local and distortional buckling test on lipped channel and z section using FE based software ABAQUS. These models were experimentally validated and showed that FEM can be accounted for the complex material and geometrical nonlinear behaviour.

Research findings and gaps

By reviewing the characteristics, design methods, numerical methods and range of literature on the flexural behaviour of CFS Z sections beams. Some important conclusions drawn are summarised as follows:

- Material properties, initial imperfections, residual stresses, and nonlinearities have an effect on the flexural behaviour of CFS members.
- The majority of studies were generally focused on the instability phenomena such as local, distortional and lateral-torsional buckling of CFS beams of individual sections.
- There are limited studies on the flexural behaviour of CFS Z section beams subjected to bending in the literature.
- DSM and EWM, which are the design methods for CFS sections in different codes, provide ultimate limit design and do not consider serviceability in the design of CFS beam.
- Currently, design manuals for CFS members are based on classical beam theory, which is linear analysis. However, in the linear analysis, the calculation of deflection is still very complicated due complex geometry of CFS sections.
- It is suggested by literature that a completely nonlinear computational model may be the ideal solution in the prolonged structural analysis and future development of the performance based design methods of CFS members.
- Numerical methods based on FEA have been widely used in performance-based analysis of CFS members, which have all shown the successful application in verifying CFS beams with the high level of accuracy and economic when compared to tedious experiments.
- The accuracy of Finite element model depends on the elements, finite mesh, application of load and boundary conditions.

Therefore, these findings and the limited research support the topic of this project. It would be valuable to conduct research on Z section using FEA and to determine whether or not currently adopted analysis is correct so that performance of Z section can be enhanced.

Numerical modelling

Numerous numerical software is available to predict the structural behaviour of CFS members subjected to different actions. Most of them use FEA to predict accurate results provided that the loading and boundary conditions and relevant mechanical properties are modelled correctly. Finite element modelling is an important tool in the structural engineering that can eliminate the extensive physical resources and time requirements in experimental investigations.

In this study, ANSYS Mechanical APDL software is used in the nonlinear FEA of CFS Z section member to study the effects of sectional deformation subjected to bending. In this section, FE models are developed for five different profiles of Z section beam considering geometric nonlinearity for analysing flexural behaviour. The model is established through the following stages:

1. The geometric model generation using suitable element
2. Assigning the material properties and discretization of geometry into meshes
3. Applying the boundary conditions and loading
4. Lastly, solution type employed to derive non-linear behaviour of members.

Geometry

The geometry of a model in ANSYS is generally created by direct generation, creating a solid model within ANSYS or by importing a solid model created in separate CAD software. A simple geometry is generated directly within ANSYS, as it provides much more control over the numbering of the nodes and elements. However, for comparatively large or complex model this methodology becomes less flexible. A total of five different geometry of Z section beam (see Figure 2) of length L, web depth h, flange width b, lip length c and thickness t were generated directly in ANSYS to evaluate load-deformation characteristics in different beam dimensions and thickness. Single span purlin load tables (Limited, 2010) were used for Z section dimension and working span load (UDL), calculated in accordance with BS5950 Part 5 in conjunction with full testing. Albion Sections zed purlins are manufactured by cold roll forming pre-hot dipped galvanised steel S450, having a guaranteed minimum yield strength of 450N/mm2 and Z275 galvanised coating (Limited, 2010).

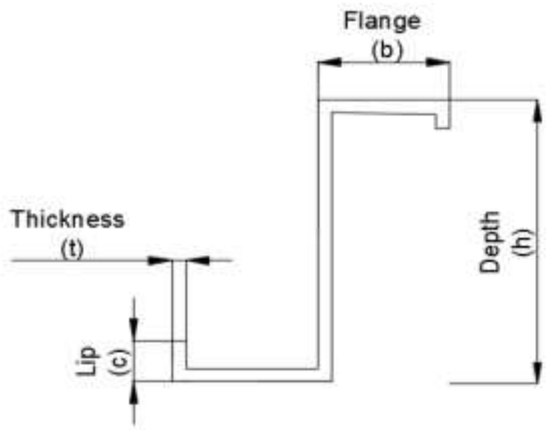

Figure 2: CFS Z section geometry

The load table including dimensions, span and thickness for sections adopted is presented below:

Table 1: Z section specimen: dimensions and load (Albion Section properties and load tables, 2010)

Section (reference)	Depth (mm)	Flange (mm)	Lips (mm)	Thickness (mm)	Span (m)	UDL (kN)
Z20620	200	65	20	2.0	6.5	10.33
Z20625	200	65	20	2.5	7.0	10.88
Z24625	240	65	20	2.5	6.5	19.84
Z24630	240	65	20	3.0	8.0	15.11
Z30730	300	75	20	3.0	9.0	22.03

Element type

The accuracy of the FEA results highly depends on the selection of appropriate element to predict the real behaviour of the member. ANSYS program has large a variety of element types available. These elements are mainly classified on the basis of features embedded into element type. Link, plane, beam, solid and shell are common types of elements used in the structural analysis. In this study, all profiles of Z sections are modelled using the SHELL181 element as CFS member are made from thin steel sheets. The element is defined as 4 node shell element, each node having 6 degrees of freedom, 3(x, y and z) in the translational direction and 3 in rotational directions. ANSYS (2010) has listed this type of element to provide good results in modelling CFS members, having both geometrical and material nonlinearities. SHELL181 has features, which can represent CFS members effectively, as it is suitable for analysing thin to moderately thick shell structures. This element is capable of describing large-deflection and large strains nonlinear applications, as its formulation are based on logarithmic strain and actual stress measures. It can account for changes in shell thickness during nonlinear analysis and uses linear interpolation. ANSYS uses five points of integration through the thickness of the shell for all options. SHELL181 type element uses reduced (lower order) integration to form element stiffness; reduced integration lowers the amount of time required for the nonlinear analysis of the model giving better performance.

Material properties

The material properties are one of the most important parameters in the FEA of CFS sections, because the material model represents a mathematical relationship between response and load. It also requires input parameters, so that model matches the material behaviour. The material properties of the CFS Z section are assumed to be linear elastic, i.e. the response is the stresses that are directly proportional to the strains and the material will fully recover the original shape when unloaded. All the sections modelled were based on isotropic strain-hardening behaviour. The stress-strain relationship of CFS members was earlier described by a gradual yielding behaviour followed by a considerable period of strain hardening. The isotropic strain-hardening behaviour is accurate compared to perfect plasticity behaviour, as it can simulate the actual mechanical properties of CFS. Therefore, sections were modelled as elastic with Young's modulus E equal to 210 GPa and Poisson ratio PRXY equal to 0.3. Initial imperfections and residual stresses were not considered, due to lack of knowledge on the production history for each profile.

Finite element mesh

The size of the element (mesh) is also an important parameter in the FEA that influences the results. The accuracy, convergence and speed of solution depend on the mesh size. Schafer et al. (2010) modelled a typical CFS channel section, using ABAQUS for a range of meshes from coarse to fine to demonstrate the mesh sensitivity. A finer mesh was found to predict more accurate results, as the peak and collapse responses by adopting coarse mesh resulted in inaccuracies. However, it requires more disk usage and processing time. Hence, the excessively fine mesh is not the economical simulation. Therefore, preliminary analyses were carried out to find the optimum size of the finite element mesh. It showed that variation of deflection of Z section beam corresponding to the element size less 10mm mesh is negligible. Consequently, the area of all five specimens was modelled using 10mm finite element mesh using free mesh technique with having objectives of creating a sufficiently fine mesh to model the essential features of the deformed shape, and to minimize the number of elements to reduce computation time. Figure 3 shows the Z section typical meshes used in the analysis.

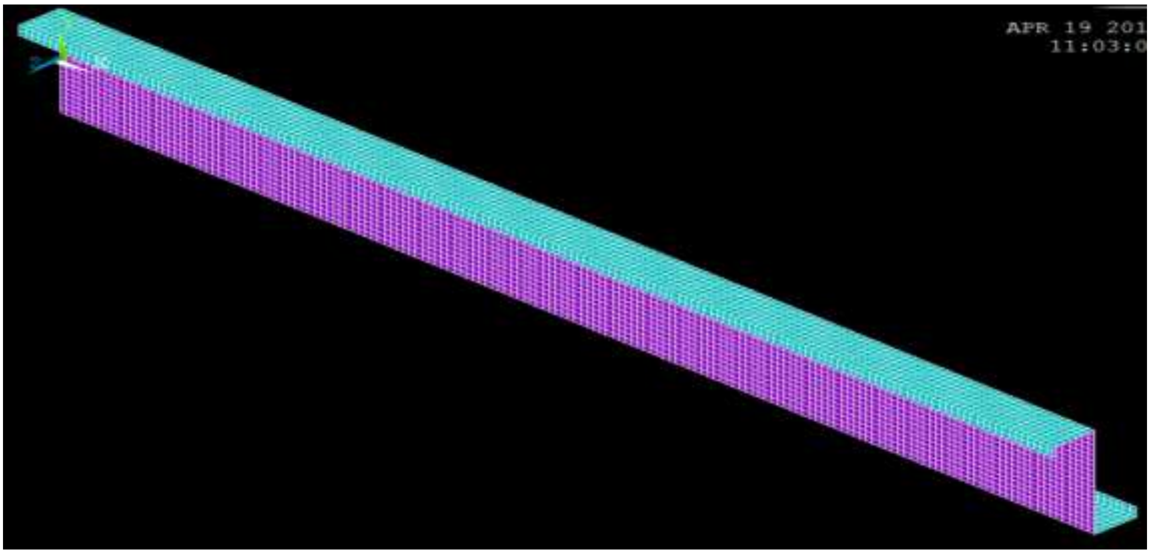

Figure 3: Meshing of Z section in ANSYS modelling

Boundary conditions and loadings

In this paper, FEA is carried out to investigate the sectional deformation of the Z section under bending, where applied boundary condition and loading on section are most principal parameters that can influence output deflections. Hence, accurate modelling of these conditions is vital. There are several ways of applying these entities to a model: as displacement, structural force or pressures onto either directly on geometric units or a set of nodes in the finite element model considering a pure structural analysis. Boundary conditions are expressed based on supports, connections, and contacts of structures. In reality, boundary conditions can change their status during the loading affecting the final solution. These effects are difficult to simulate in a finite element model, and therefore, are often neglected. Simply supported boundaries are considered herein. With respect to loadings, geometry and displacement constraints many structures exhibit one or more symmetries. The size of the model can be optimized by employing suitable boundary conditions along the planes of symmetry by identifying symmetric characteristics. Since the applied loads to the beam-section is symmetric relative to the plane of symmetry. Therefore, half span of CFS beam is modelled by applying symmetric boundary condition to symmetrical half end (mid span) in order to reduce the number of elements and nodes as well computational time. Symmetric boundary condition implies that the displacements normal to the plane of symmetry and rotations about the axes in the plane of symmetry are zero at the plane of symmetry. Typically, applying symmetry in model yields better results as it leads to a finer, more detailed model than would otherwise be possible (ANSYS, 2010).

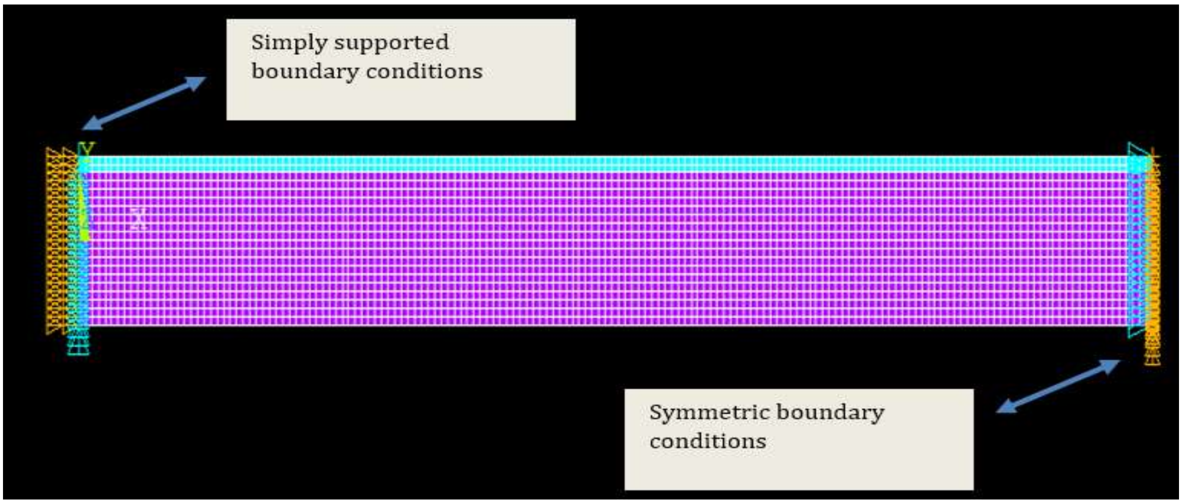

Figure 4: Application of boundary in ANSYS modelling

The displacement boundary conditions are applied to all nodes at both ends of the beam, as shown Figure 4. On the simply supported end, the nodes along the boundary of Z beam cross section are constrained such that the displacement to Y-axis and Z-axis and rotations about X-axis are zero (u_y=0, u_z=0 and θ_x=0), while the symmetric boundary conditions are applied to the perimeter nodes at another end surface such that rotation about Y-axis and Z-axis and displacement along X-axis are zero (θz=0, θy=0, and u_x=0). A uniform work-load referred from Albion Section Properties and Load Tables (2010) is applied as force vertically downward on the nodes at the top of purlin web as shown in Figure 5.

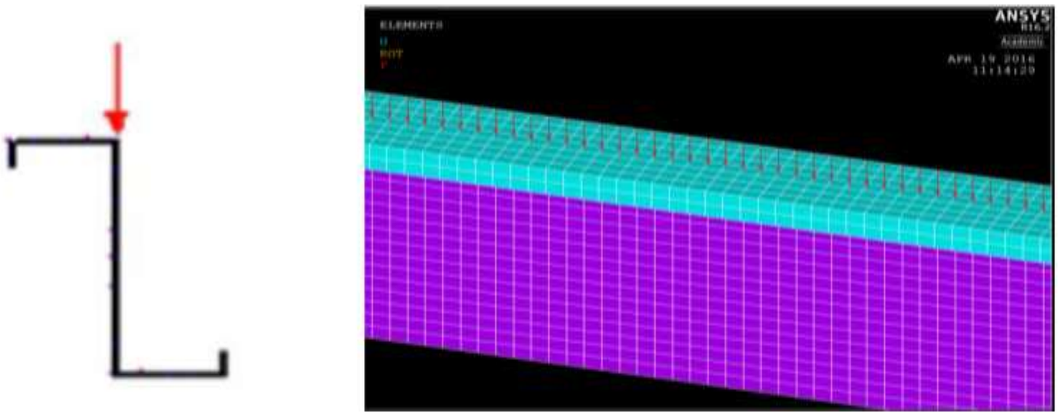

Figure 5: Application of load (red arrow) in ANSYS modelling

Solution type

During the solution phase of the analysis, ANSYS program performs a number of numerical actions to solve simultaneous equations that the finite element method generates and corresponding results in terms of nodal values are stored to database.

There are several methods of solving the simultaneous equations available in the ANSYS depending on the analysis type. The type of analysis can be static, transient dynamic, harmonic, modal and buckling depending on the behaviour of the structure being studied and therefore, must be stated at the beginning of the solution phase.

A large displacement static analysis considering geometric nonlinearities was undertaken to determine the nonlinear behaviour of CFS Z section beam under bending. ANSYS employs the Newton-Raphson method to solve nonlinear behaviour of the structure. Before each solution, the Newton-Raphson algorithm uses the tangent modulus stiffness corresponding to the previous iteration to locate the next deformed position on the structure. The stiffness matrix continues to be updated until the difference the applied load and the projected position is within some acceptable tolerance. In this approach, the load is subdivided into a series of load increments, which are applied over several load steps. Each load step is further divided into substeps or time steps so that ANSYS program can apply the specified load gradually and to obtain the accurate results. The number of substeps can be defined in ANSYS using "Solution Control" dialogue box to apply the load gradually. Hence, the load was applied in one step with 20 initial substeps. The number of iterations required to converge problem at this substep is monitored by default in ANSYS program. If convergence cannot be achieved, then program will automatically reduce the load increment it takes and adds more time stepping with automatic time stepping on option. Using the Newton-Raphson method may lead the tangent stiffness matrix to become singular or non-unique, which can cause severe difficulties in convergence. For such conditions, the 'arc-length' method was activated in this study as an alternative iteration technique to help avoid bifurcation points and track unloading in convergence problems. The 'arch–length' method makes the static equilibrium iterations to converge along an arc even when the slope of the load-deflection curve becomes zero or negative thus preventing any divergence issues (ANSYS, 2010).

Accordingly, nonlinear analysis can be summarized into three fundamental operations:

1. The top one consists of the load steps which are defined explicitly over a time span and within these steps loads are assumed to vary linearly.
2. To apply the load gradually, substeps or time steps are directed into the program to perform several solutions within each load step.
3. Finally, a converged solution is obtained by performing a series of equilibrium iterations at each substep (ANSYS, 2010).

Validation of model

It is essential to validate the finite element model developed to simulate the structural behaviour of CFS members. For this purpose, I section beam was modelled using same procedure as adopted in this study to check the accuracy of the model. The dimensions used were similar to the section Z20620, i.e. 6.5m length, 65 mm flange, 200mm depth and 2mm thickness. The maximum displacement was obtained at mid-span using nonlinear FEA and was validated against linear displacement, which was calculated using maximum deflection from full length UDL equation. Good correlation was found to be between the two solutions. Therefore, nonlinear finite element modelling and procedure adopted in this study are accurate.

Results and discussion

After building the model and performing the nonlinear analysis, the results of the analysis were obtained by using ANSYS POST1; the general postprocessor, and POST26; the time-history processor. These processors are inbuilt in the ANSYS program and used for the reviewing the results of the analysis in different modes. POST1 was used to review results of the analysis over the entire model at specific load steps and substeps, to obtain the various nodal stress and strain values and the deformed shape. POST26 was used to provide the variation of results at specific points in the model over the time history which is one of the most important step in the analysis, as the load vs deformation curve at mid-span of the Zed section under bending were obtained using this processor.

The half-length models of five Z section profiles were modelled and nonlinear static analysis was performed. A uniform work-load specified in load tables (Limited, 2010) was applied in terms of displacement on the nodes, at the top of the web along the length of the beam and calculated against the displacement. The result was obtained in terms of nodal displacement calculated at each node and load in terms of time factor at each substep or time step. The nodal displacements and applied load for corresponding substep were obtained for all profiles at mid-span.

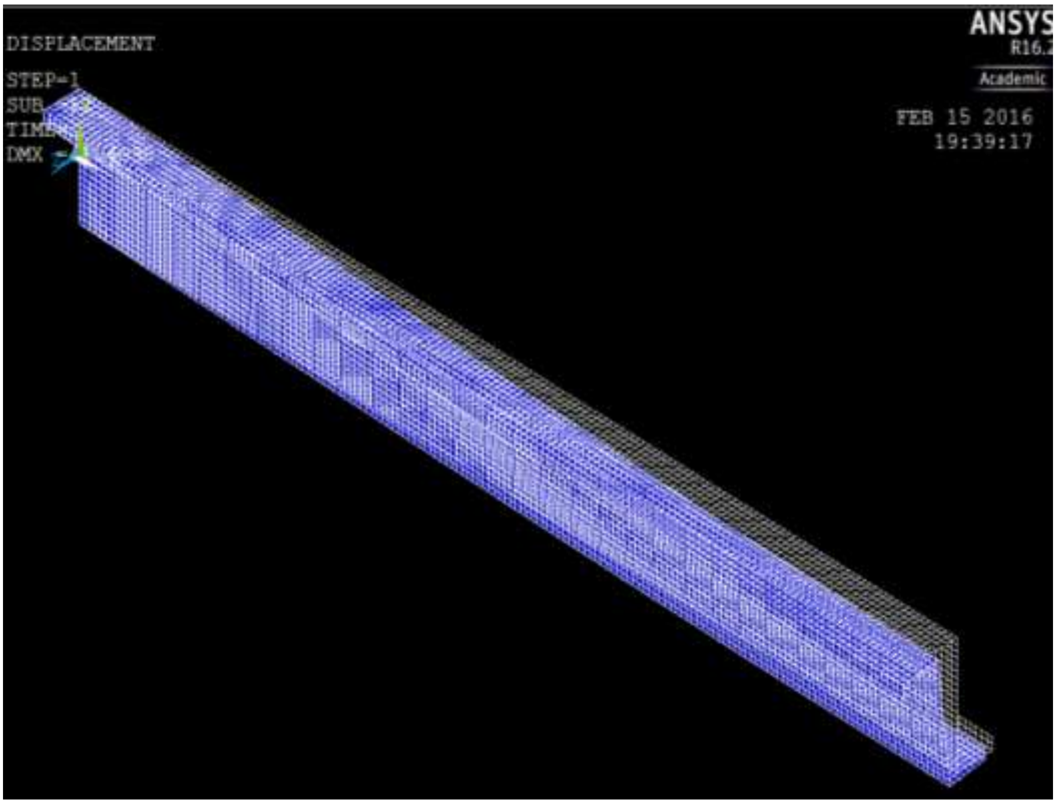

Figure 6: Deformed shape (blue) of Z section (Z20620) purlin with undeformed shape at the maximum loading

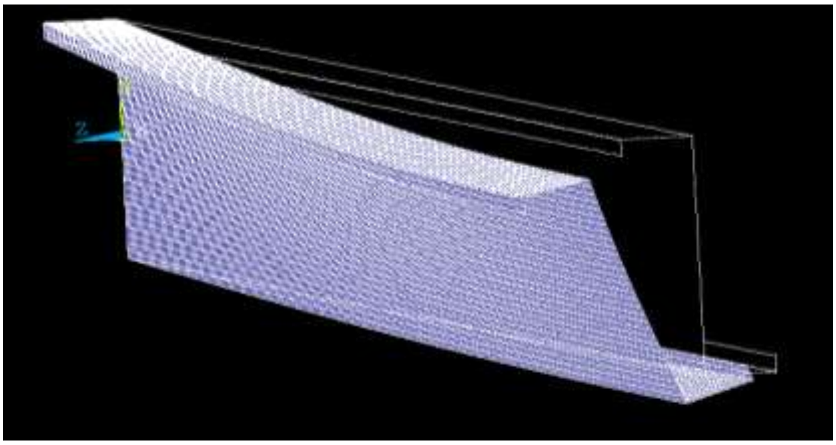

Figure 7: Deformed shape of section Z30730 with undeformed edges

The aim of this project was to study the effects of sectional deformation on the performance of CFS Z section profiles, having different size and thickness under bending due to uniform loading. The analysis was performed using the finite element based program ANSYS to check the accuracy of currently adopted linear analysis for calculation of deflection these section in all design manuals. As part of this study, five section profile with four different web depth and their thickness ranging from 2 mm to 3 mm were modelled using half of their length given Albion section properties and load table guide developed in accordance with British Standard. Figure 6 represents the deformed shape (blue meshed area) of Z section (Z20620) purlin with the undeformed shape at the maximum loading i.e. last substep. It is observed that maximum deformation occurs at the line of symmetry, which is mid-span as only half-length was modelled considering the symmetry. Therefore, it suggests that boundary conditions and loading were applied correctly. In addition, from Figure 7 is evident that there has been a torsion effect or twisting during bending of CFS sections which result in loss of bending stiffness of cross-section as the CFS section used in this study are slender elements. However, the linear analysis assumes that stiffness of section remains constant and section displaces only in transverse and lateral direction without twisting during bending and section does not change as shown in Figure 7. So, in order to take this effect into account nonlinearly of solutions should be considered while calculating the deflections.

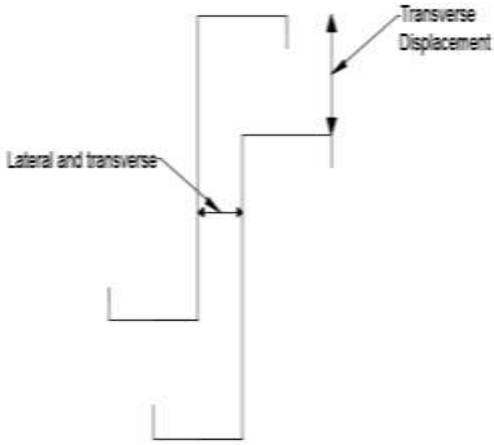

Figure 8: Displacement of Z section during linear bending

Using the tables of nodal displacement and load corresponding to its substep, load vs. deflection curves at mid-span of web were plotted in order to study the behavior of CFS z section profile with different dimension and thickness under bending. The resultant graphs for five zed section profile are presented in Figure 9-13.

In all graphs (Figure 9-13) below, the solid blue line represents the nonlinear solution for deflection at mid-span was obtained using nonlinear analysis results tables while red dashed line denotes, the linear solution was plotted on the basis of the principle of superposition as explained in earlier in the nonlinear analysis section. The red dots in plots signify the linearity and were extend to form the gradient of linear solutions.

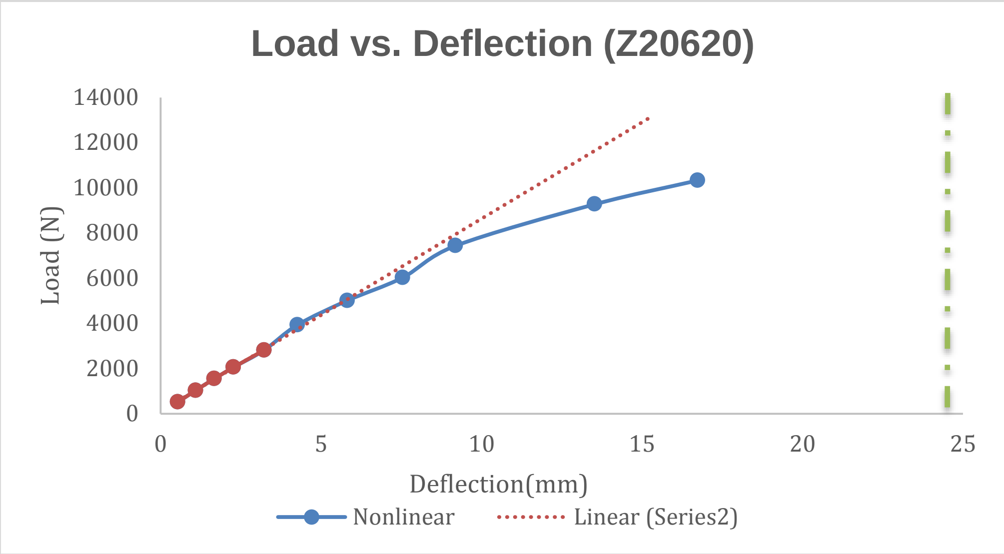

Figure 9: Load vs. deflection curve for section Z20620 at mid-span

The dot-dashed green line symbolises, maximum deflection limit calculated based on serviceability limit state (≤L/250).

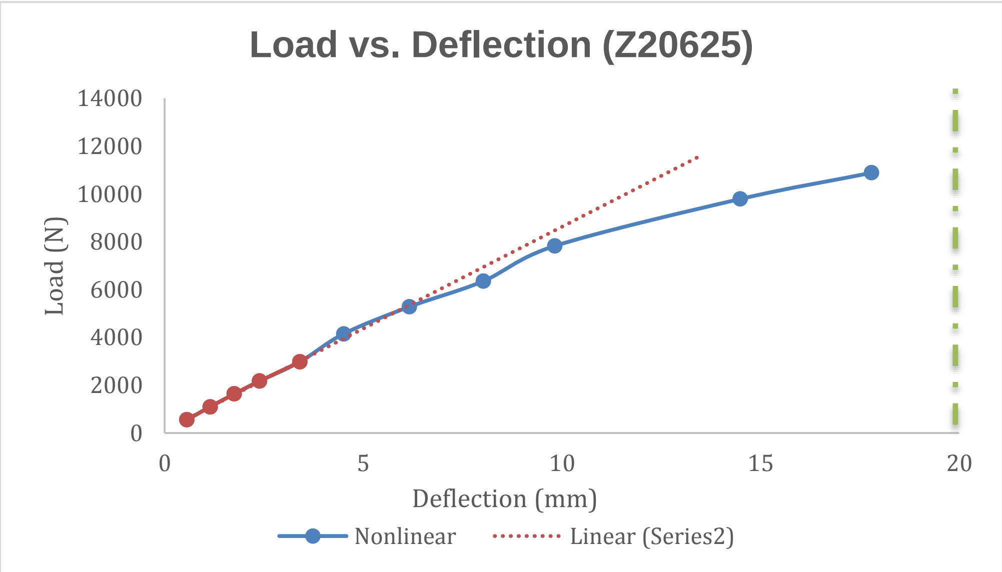

Figure 10: Load vs. deflection curve for section Z20625 at mid-span

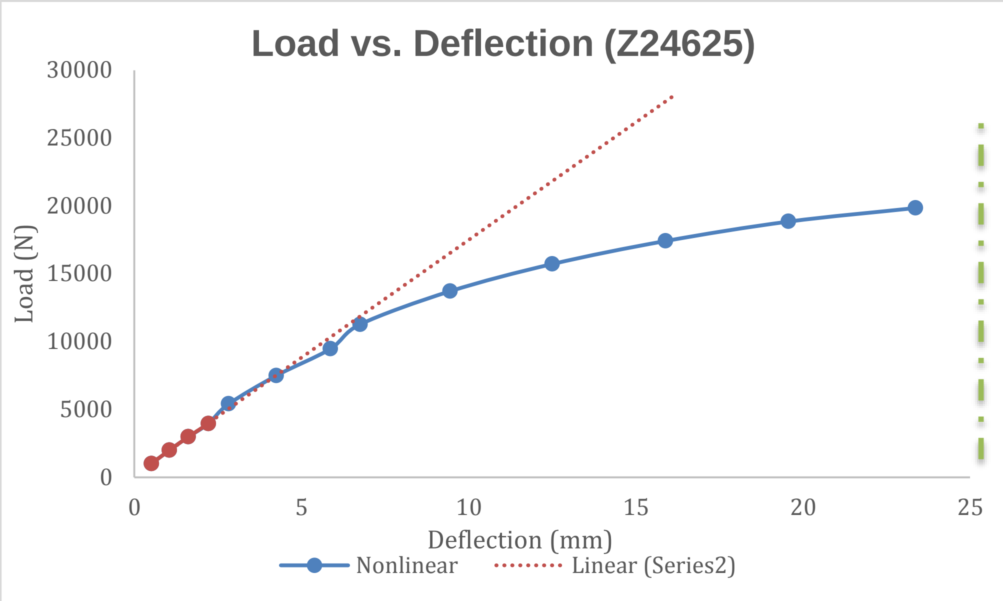

Figure 11: Load vs. deflection curve for section Z24625 at mid-span

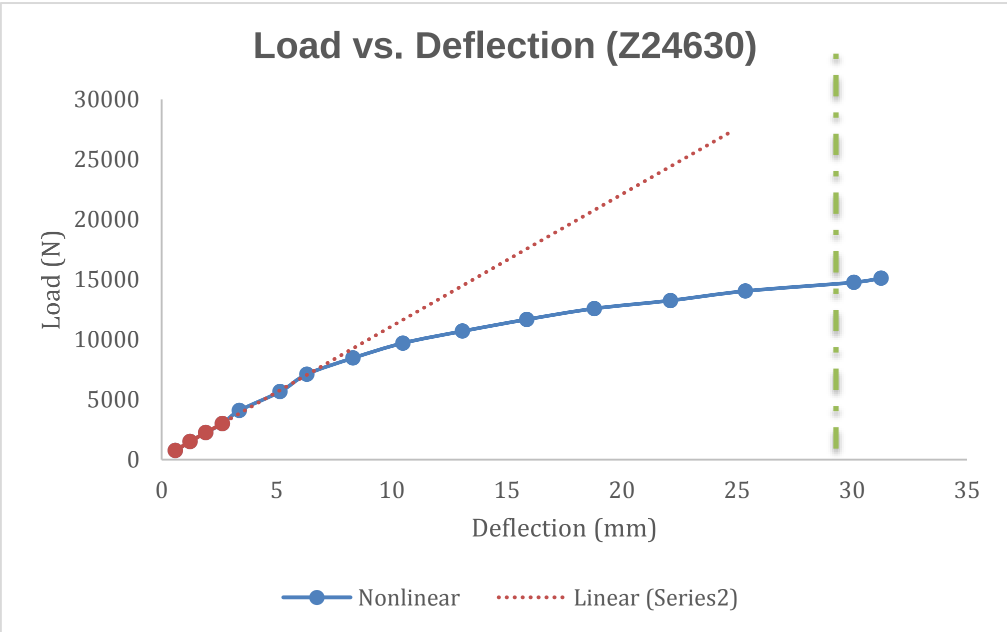

Figure 12: Load vs. deflection curve for section Z24630 at mid-span

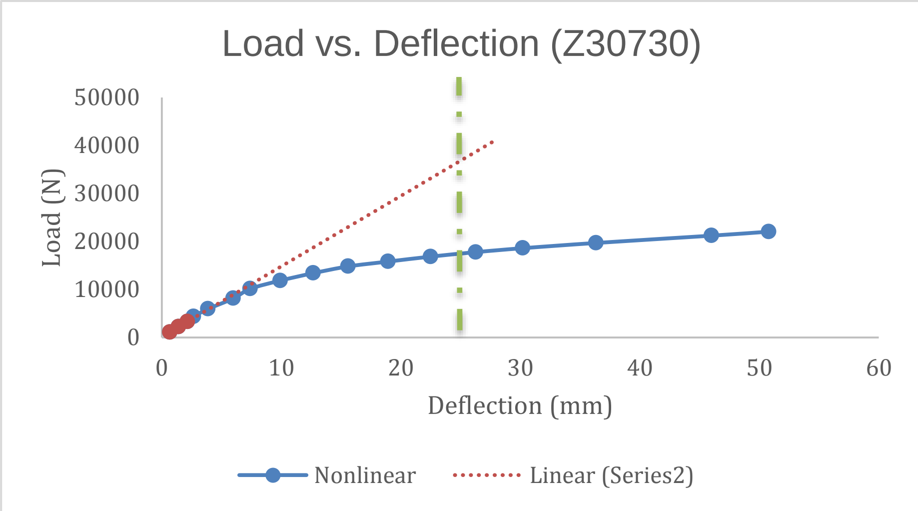

Figure 13: Load vs. deflection curve for section Z30730 at mid-span

All of these load vs deflection curves at mid-span clearly demonstrate reduction of stiffness during bending of zed section. For instance, in Figure 9 the gradient of the plot represents the stiffness of section. It was observed that relationship between loading and displacement is linear for the initial portion (in red dots), the stiffness of cross section remains constant and as loading increases deviation in slope starts appearing results in the reduction of stiffness. A Similar pattern was spotted for rest sections where stiffness reduces with the increase in loading. Hence, it could conceivably be hypothesized that for small loading section does not change. These results provide further support for the hypothesis that a link may exist between applied load and nonlinearity. It was observed from plots of load-displacement that for the load less than 4 kN section does not change i.e. the relation between load and displacement remains linear and for load greater than 4 kN, gradient decreases, and stiffness start reducing. In other words, the transition from linear to nonlinear (curvature) occur after 4 kN load for all the sections analysed. Therefore, the nonlinear solution needs to be considered for load greater than 4 kN for CFS Z section beams.

Table 2: Percentage difference in linear and serviceability predicted maximum displacement in comparison with nonlinear FEA maximum displacement at mid-span for all Z Section specimens analysed

Section	Depth (mm)	Flange (mm)	Lips (mm)	Thickness (mm)	Linear difference	Serviceability difference
Z20620	200	65	20	2.0	28 %	22 %
Z20625	200	65	20	2.5	27 %	22 %
Z24625	240	65	20	2.5	51 %	27 %
Z24630	240	65	20	3.0	55 %	49 %
Z30730	300	75	20	3.0	69 %	65 %

Above all, it is interesting to note from Table 2 and Figure 14 that the difference in maximum displacement at mid-span for both linear solutions and serviceability predictions (≤L/250) is significant for structural considerations when compared nonlinear FEA obtained solutions for all analysed specimens. This clearly demonstrates that why there is need of considering nonlinear analysis. For section Z20620 with dimensions: h 200 mm, b 65 mm, c 20 mm and 2 mm of thickness t, the difference between maximum deflection of linear solutions was found to be 28% whereas for section Z20625 which has same dimensions but with 2.5 mm thickness difference was found to be 27%. Likewise, for sections Z24625 and Z24625 with the same dimension of h 240 mm, b 65 mm and c 20 mm but with different of thickness 2.5m and 3 mm. A similar trend in linear solution difference was found to be 51% and 55% respectively. Finally, a difference of 69% was found for section Z30730 with h 300 mm, b 75 mm, c 20 mm and t 3 mm. These results can be explained in part by the proximity of thickness and web depth as the size of flanges and lips remains same for all four sections. The increase in thickness of section does not affect the solutions significantly but deeper web results in almost double difference in both solutions. It is, therefore, likely that such connection exists between web depth and difference between linear and nonlinear solutions. For the structural application, the section depths typically range from 100 mm to 400 mm, thus on the basis of above observations, the nonlinear solution becomes more significant for the section with web depth 200 mm or greater. Hence, based on the FEA evidence, design manuals should consider nonlinear solution for deflection calculation of CFS members.

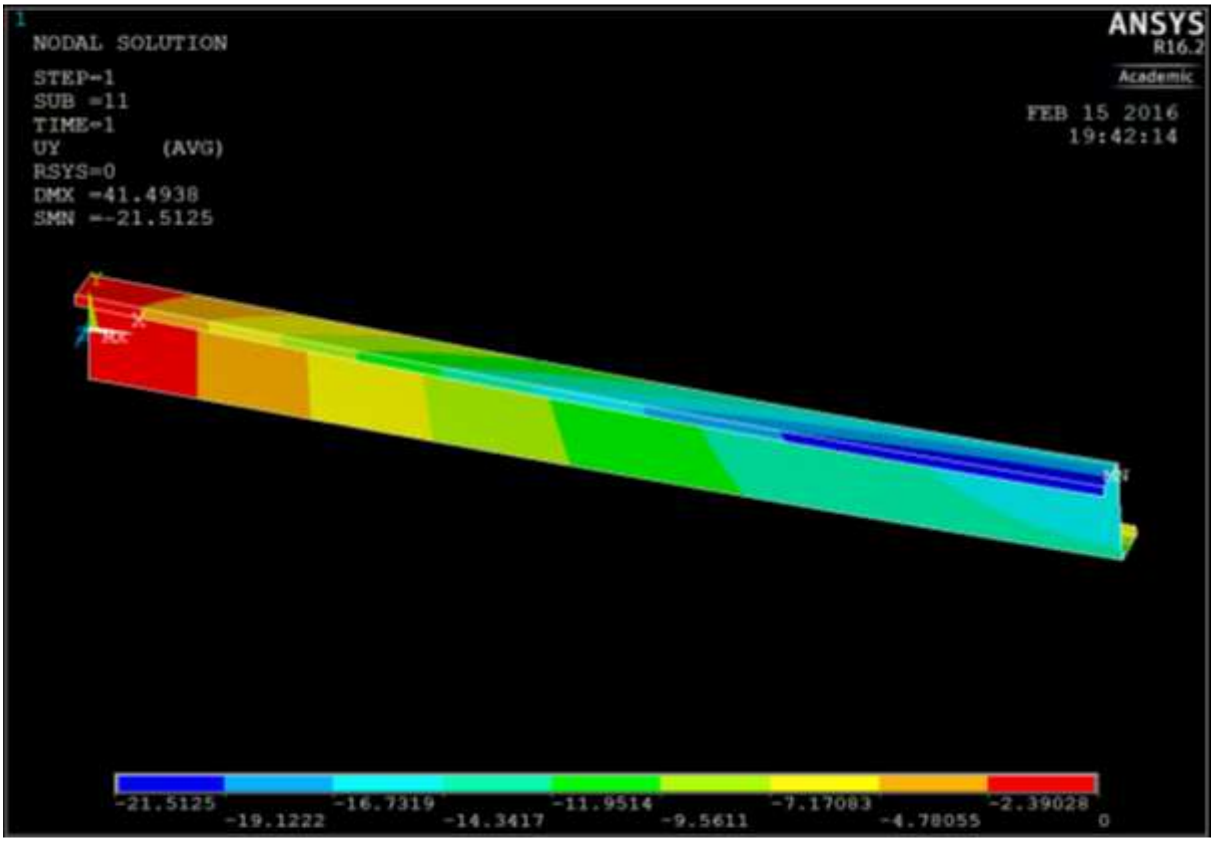

Figure 14: Difference of linear and serviceability displacement in comparison with nonlinear displacements

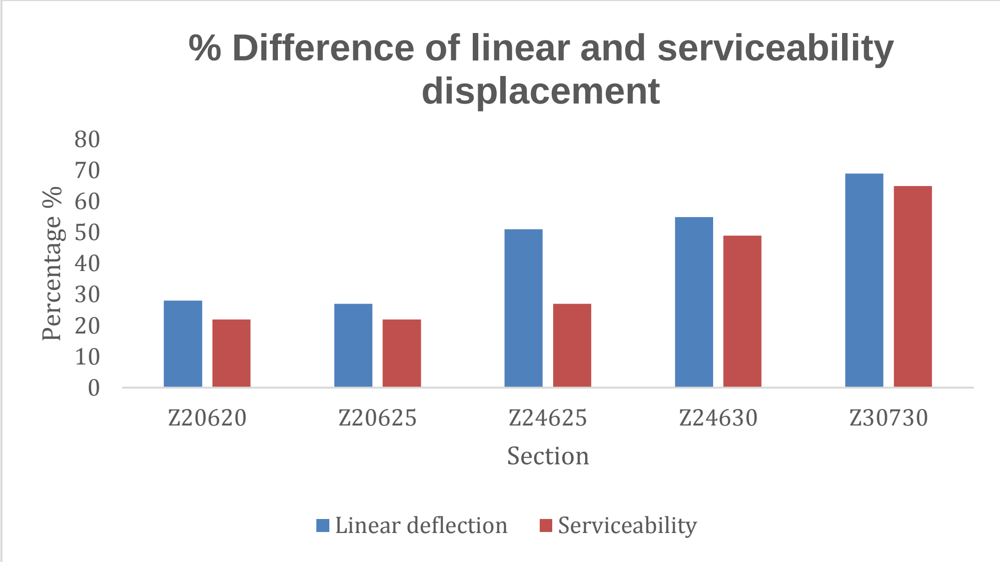

Figure 15: Contour plot of displacement variation across section Z20620 in deformed state

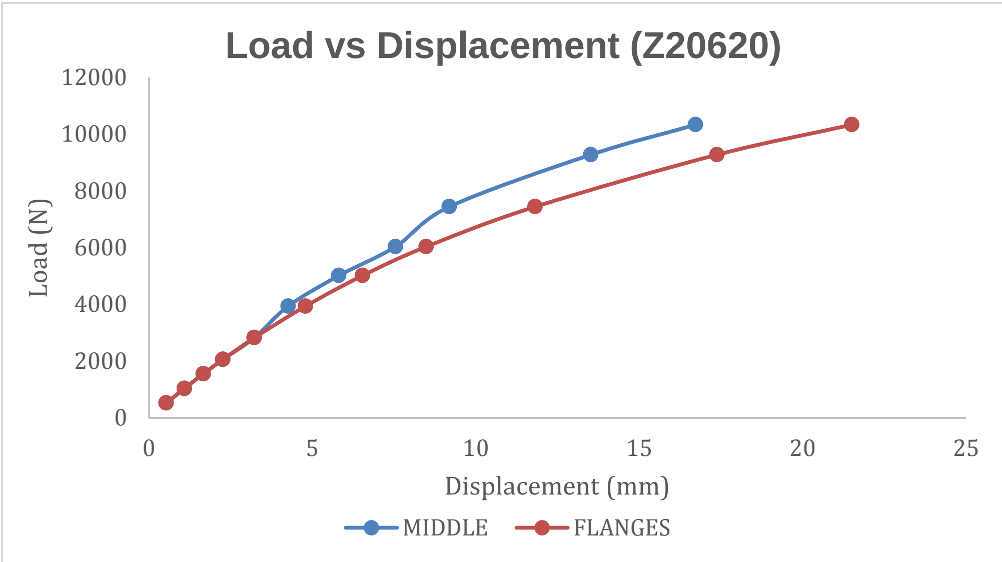

Figure 16: Load vs. deflection curve for section Z20620 at mid-span for web and flanges

Alternatively, the significance of nonlinear analysis can be explained by Figure 15, which illustrates the displacement variation across the beam in the deformed state. It is important to note that displacement at mid-span varies for lip, flanges and the web whereas linear analyses assume that displacement should be same for web, lips and flanges, since loading was applied at web line. Additionally, from Figure 16 it was noted that difference in maximum displacement between the web (middle) and flanges is 22%, which significant to ensure the safety of structure where lips are used to improve section efficiency under compressive loads. In particular, the finite element method adopted herein is necessary but sufficient for understanding fully nonlinear analysis for the CFS member performance under bending. Such a nonlinear analysis model will also need to incorporate initial imperfections and residual stresses so that it may provide engineers with a realistic prediction of deformation characteristics that can be used in design.

Conclusion

This paper presented a numerical investigation on the performance of CFS Z purlins under bending subjected to uniformly distributed loads applied at the shear centre of the section. Currently, in all design codes deflection of CFS Z section is calculated based on classical bending theory, mostly because of the complexity of CFS cross-sections and it is assumed that load-displacement characteristics are linear, and stiffness of section does not change with loadings. However, CFS sections during bending develop a tendency to twist results in the reduction of stiffness, due to their slenderness, which is currently not accounted during deflection calculations. Therefore, in order to check the accuracy of linear solutions, an accurate FEA model was developed using ANSYS program. However, finite element modelling in literature has been shown to have the potential to provide a more fundamental understanding of behaviour as well as the facility to produce detailed information on the response of individual CFS members. Five commercially available single span Z-purlin section having different dimensions and thickness were selected for FEA. The model utilized a 4-node isotropic shell element to represent the body of CFS profiles and only half span of the beam was modelled to save computational time. The FEA model considered geometric nonlinearity of CFS members and the load was applied in terms of load increment over several load steps using large static displacement solution.

An examination of the deformed geometry of nonlinear FEA validated that, the bending-torsion occurs in all sections analysed. The results obtained by nonlinear analysis showed a significant difference between the classical linear solution and nonlinear FEA solutions in displacement at mid-span of all five CFS Z purlin profiles. Similarly, comparison of nonlinear maximum deflection at mid-span with predicted by serviceability limit states (≤L/250) revealed significant difference as well. Furthermore, displacement variation at mid-span along the web, flange, and lip of the CFS beam was varying although load was applied at web line. The present study confirms from these finding that bending-torsion results in the reduction of stiffness and contributes additional evidence that suggests that nonlinear analysis plays a significant role in deflection calculation of CFS Z sections and on the performance of Z purlins. Hence, in the design process of CFS Z sections, the nonlinear analysis should be used to calculate the deflection.

The reduction in stiffness under bending was further illustrated using load versus deflection curves at mid-span obtained from the nonlinear analysis. For the case where the applied load is greater than 4 kN, reduction in stiffness becomes substantial and it is recommended that for loadings greater than 4 kN, the nonlinear analysis should be considered. Furthermore, by comparing the nonlinear solution of all profiles analysed it was discovered that varying the thickness does not influence the nonlinear behaviour considerably. However, with the increase in web depth of section nonlinearity increase drastically. In general, therefore, it seems that for the section with depth greater that 200 mm nonlinear solution are critical and it is recommended that nonlinear should be accounted in deflection calculation for the section with web depth of 200 mm or more.

To ensure safety and prove the reliability of a CFS Z section member, it is necessary that member is analysed nonlinearly considering other sources of nonlinearities, residual stresses and initial imperfections as literature states that these have an influence on the performance of CFS members, before the cross-section is adopted in the industry. FEA observations and results achieved from this study can be used further developing finite element models and to notify design codes and analytical studies such that they can provide a better way to assess serviceability and large deformation effects for CFS members.

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